Student Number

Hunters Hill High School

2020 Trial Examination

Mathematics Extension 1

General Instructions

- Reading time – 10 minutes
- Working time – 2 hours
- Write using black pen
- Calculators approved by NESA may be used
- A reference sheet is provided at the back of this paper
- In Questions 11-14, show relevant mathematical reasoning and/or calculations

Total Marks: 70

Section I – 10 marks (pages 3-6)
Attempt all Questions 1–10
Allow about 15 minutes for this section

Section II – 60 marks (pages 7-12)
Attempt Questions 11–14
Allow about 1 hour and 45 minutes for this section

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Section I

10 marks
Attempt Questions 1–10
Allow about 15 minutes for this section

Use the multiple-choice answer sheet for Question 1–10.

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1. In how many way can the letters of the word SUCCESS be arranged?

(A) $7!$

(B) $\frac{7!}{2!}$

(C) $\frac{7!}{2!\,3!}$

(D) $4!$

2. The parametric equations $x = ct, y = \frac{c}{t}$ form a

(A) Line

(B) Parabola

(C) Circle

(D) Rectangular Hyperbola

3. Which if the following vectors is perpendicular to $\begin{bmatrix} -2 \\ 3 \end{bmatrix}$?

(A) $\begin{bmatrix} -2 \\ 3 \end{bmatrix}$

(B) $\begin{bmatrix} -3 \\ 2 \end{bmatrix}$

(C) $\begin{bmatrix} -6 \\ -4 \end{bmatrix}$

(D) $\begin{bmatrix} -4 \\ 6 \end{bmatrix}$

4. The parametric form of a function is given as $y = \sin t + 1,\ x = \cos t$.
The Cartesian form of the function is

(A) $y = x + 1$

(B) $x^2 = y^2$

(C) $x^2 + (y-1)^2 = 1$

(D) $\tan t = \dfrac{y-1}{x}$

5. The quadratic equation $3x^2 - 5x - 6 = 0$ has roots of α and β.
The value for the expression $\alpha^2 + \beta^2$ is

(A) $\dfrac{5}{3}$

(B) $\dfrac{61}{9}$

(C) $\dfrac{-11}{9}$

(D) -2

6. Which of the following expressions is equivalent to $2\sin 5\theta \cos 3\theta$

(A) $\sin 8\theta + \sin 2\theta$

(B) $\cos 8\theta + \cos 2\theta$

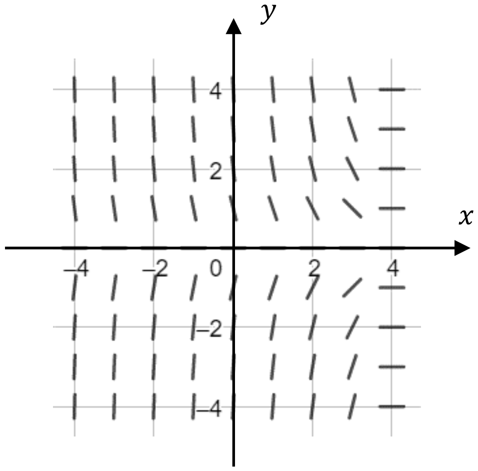
(C) $\sin 8\theta - \sin 2\theta$

(D) $\cos 8\theta - \cos 2\theta$

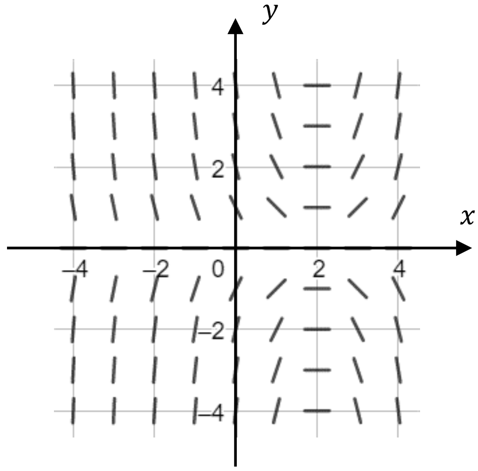
7. A differential equation is given by $y' = xy - 2y$

Which of the following slope fields best represents the differential equation?

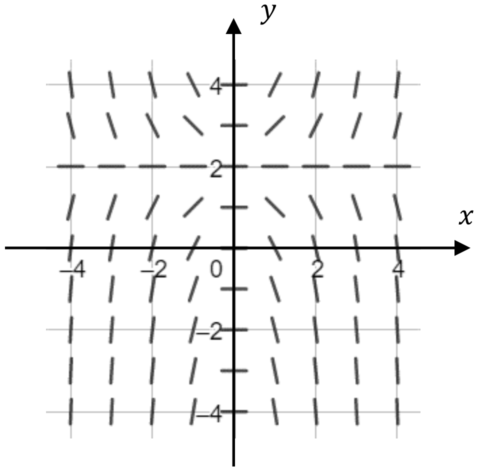
(A)



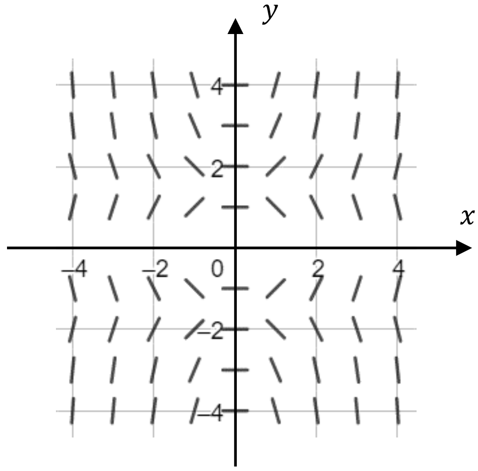
(B)



(C)



(D)



8. If A and B are the points $(-1, 4)$ and $(3, 7)$ respectively, then the unit vector in the direction of $\overrightarrow{AB}$ is

(A) $\frac{2}{5}\underset{\sim}{i} + \frac{11}{5}\underset{\sim}{j}$

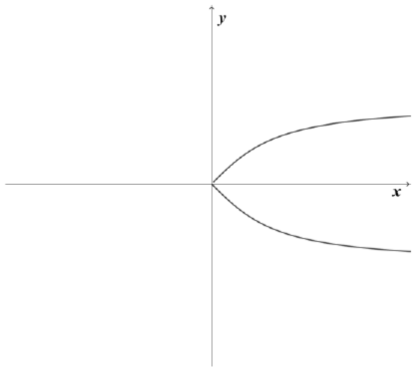
(B) $4\underset{\sim}{i} + 3\underset{\sim}{j}$

(C) $-3\underset{\sim}{i} + 28\underset{\sim}{j}$

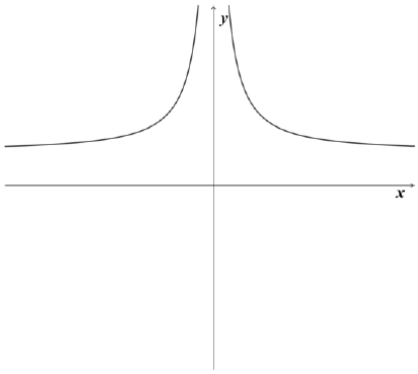
(D) $\frac{4}{5}\underset{\sim}{i} + \frac{3}{5}\underset{\sim}{j}$

9. Which of the following sketches best represents $|y| = \tan^{-1}(x)$?

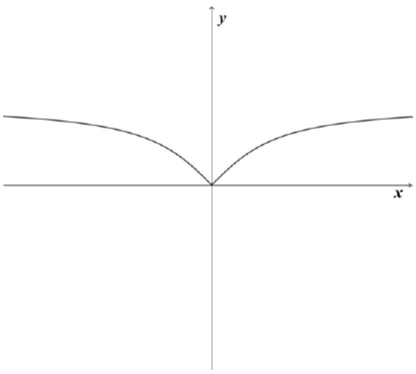
(A)



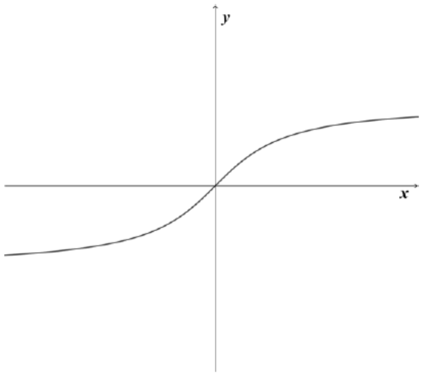
(B)



(C)



(D)



10. The letters A, B, C and D are used to form a four letter word.
How many words can be written such that D comes before A?

(A) 3

(B) 6

(C) 12

(D) 24

End of Section I

Section II

60 marks
Attempt Questions 11–14
Allow about 1 hours and 45 minutes for this section

Answer each question in a separate writing booklet.

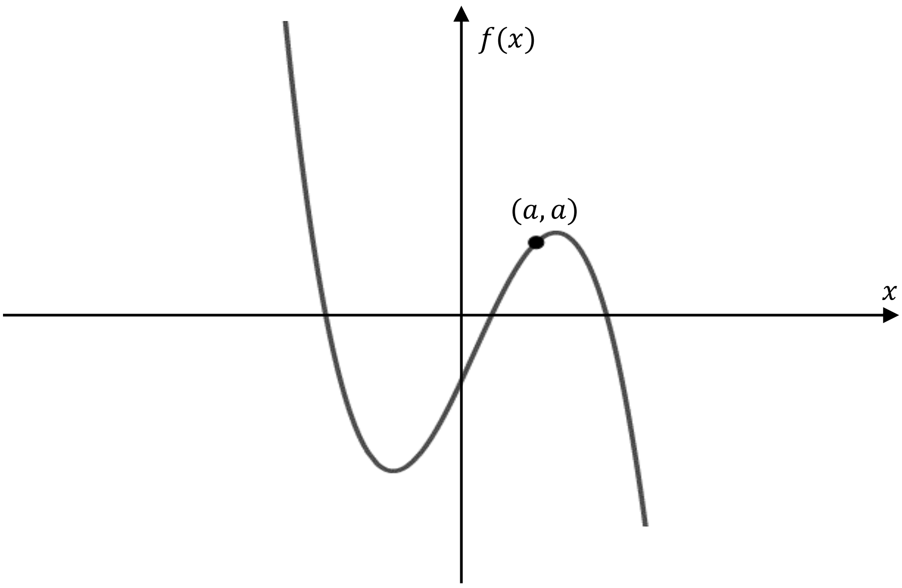
In Questions 11–14, your responses should include relevant mathematical reasoning and/or calculations.

Question 11 (15 marks) Begin a new Writing Booklet

(a) Solve $\dfrac{x}{x-1} < 3$. **2**

(b) Solve $|2x+3| \le 7$. **1**

(c) The sketch below shows the curve $y = f(x)$, with the point (a, a) shown. Copy or trace this into your writing booklet and sketch the inverse relation, $y = f^{-1}(x)$ on the same axes. **2**



Question 11 continues on next page

(d) The cubic equation $x^3 + 3Ax^2 - 4A = 0$, where $A > 0$, has roots α, β and $\alpha + \beta$.

(i) Use the sum of roots to show that $\alpha + \beta = -\frac{3}{2}A$. **1**

(ii) Use the sum of the products of pairs of roots to show that $\alpha\beta = -\frac{9}{4}A^2$. **2**

(iii) Show that $A = \frac{2}{\sqrt{3}}$. **2**

(e) Find

(i) $\displaystyle\int \frac{-1}{\sqrt{\frac{1}{4} - x^2}}\,dx$ **2**

(ii) $\displaystyle\int \sin^2 2x\,dx$ **3**

End of Question 11

Question 12 (15 marks) Begin a new Writing Booklet

(a) Using the substitution $u = \sin x$, evaluate **3**

$$\int_{\frac{\pi}{6}}^{\frac{\pi}{3}} \frac{3\cos x}{1+\sin x}\,dx$$

(b) An extended family sits down to dinner at a round table. The family consists of 5 adults, and 2 children.

Find the number of arrangements possible

(i) If there are no restrictions placed on seating. **1**

(ii) If two of the adults must sit together. **1**

(iii) If neither of the children can be seated together. **2**

(c) Consider the equation $\cos x - 2\sin x = 1$, for $-\pi \le x \le \pi$.

(i) Show that the equation can be written as $t^2 + 2t = 0$, where $t = \tan\frac{x}{2}$. **2**

(ii) Hence, solve the equation for $-\pi \le x \le \pi$, giving solutions to the 2 decimal places where necessary. **2**

(d) Water is being drained from a spout in the bottom of a cylindrical tank. According to Torricelli's Law, the volume V of water left in the tank obeys the differential equation

$$\frac{dV}{dt} = -k\sqrt{V}$$

where k is a constant.

(i) Use separation of variables to find the general solution to this equation. **1**

(ii) Suppose the tank initially holds 30L of water, which initially drains at a rate of 1.8 L/min. How long will it take for the tank to drain completely? **3**

End of Question 12

Question 13 (15 marks) Begin a new Writing Booklet

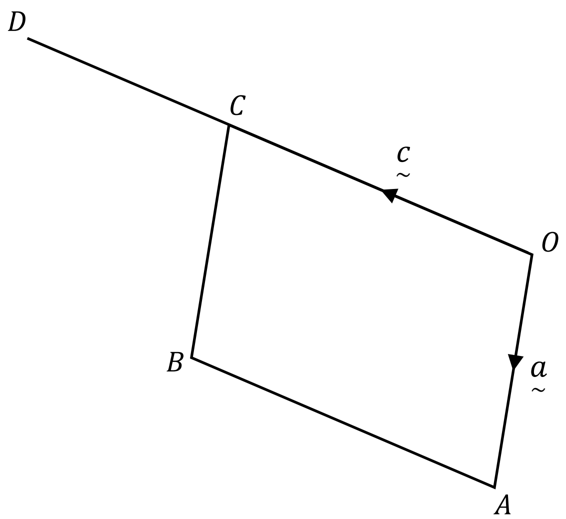
(a) The surface area of a sphere ($S = 4\pi r^2$) of radius r metres is decreasing at a rate of $0.8\ \text{m}^2/\text{s}$ at an instant when $r = 2.3$.
Calculate the rate of decrease, at this instant, of the radius of the sphere. **2**

(b) Prove by mathematical induction, $a + ar + ar^2 + \cdots + ar^{n-1} = \dfrac{a(r^n - 1)}{r - 1}$, for all integers $n \geq 1$. **3**

(c) **(i)** Prove the trigonometric identity $\cos 3\theta = 4\cos^3\theta - 3\cos\theta$. **3**

(ii) Hence, find expressions for the exact values of the solutions to the equation $8x^3 - 6x = 1$. **3**

(d) In the parallelogram $OABC$, $\overrightarrow{OA} = \underset{\sim}{a}$ and $\overrightarrow{OC} = \underset{\sim}{c}$.



X is the midpoint of the line AC, and OCD is a straight line such that the ratio $OC : CD = k : 1$.

(i) Find $\overrightarrow{OX}$ in terms of $\underset{\sim}{a}$ and $\underset{\sim}{c}$. **1**

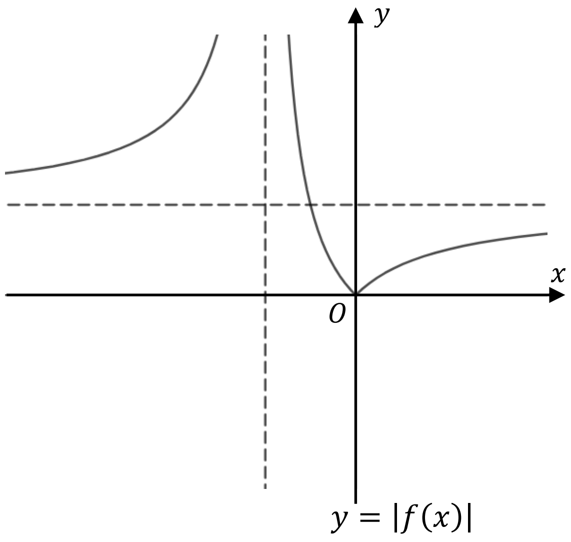
(ii) Write an expression for $\overrightarrow{OD}$. **1**

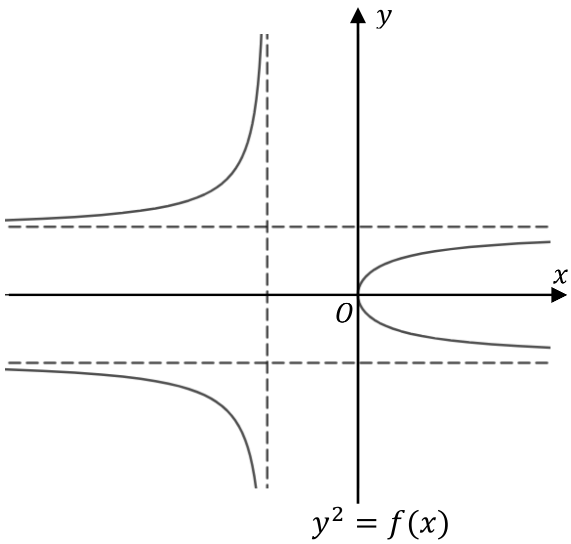
(iii) Given $\overrightarrow{XD} = 3\underset{\sim}{c} - \frac{1}{2}\underset{\sim}{a}$, find the value of k. **2**

End of Question 13

Question 14 (15 marks) Begin a new Writing Booklet

(a) The graphs of $y = |f(x)|$ and $y^2 = f(x)$ are given below.





Draw separate one-third page sketches of the following curves, clearly indicating any important features such as turning points or asymptotes.

(i) $y = f(x)$ **1**

(ii) $y = f(|x|)$ **1**

(b) Four friends go to a restaurant that serves 4 different main courses.

(i) In how many ways can the friends select meals from the courses available? **1**

(ii) If each of the friends randomly chooses which meal they have, what is the probability that exactly two of the main course options are not chosen? **2**

Question 14 continues on next page

(c) A force described by the vector $\boldsymbol{F} = \begin{pmatrix} 3 \\ -2 \end{pmatrix}$ newtons is applied to an object lying on the line l which is parallel to the vector $\begin{pmatrix} 2 \\ 5 \end{pmatrix}$

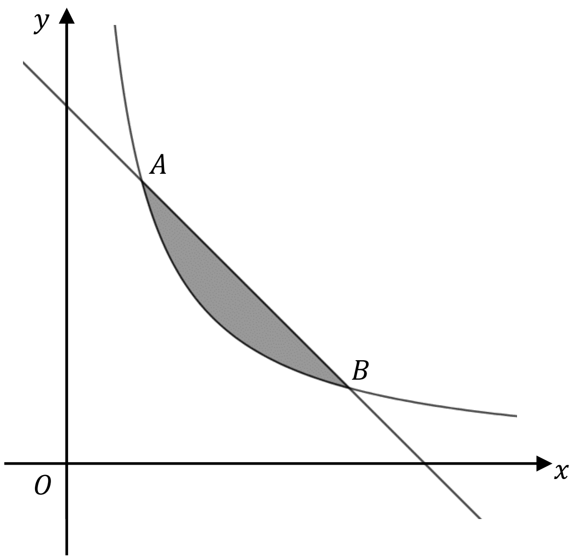
(i) Find the component of the force $\boldsymbol{F}$ in the direction of the line l. **3**

(ii) What is the component of the force perpendicular to the line l? **1**

(iii) What force parallel to the line l must be applied to the object in order for it to remain stationary? **1**

(d) The diagram below shows the graphs of $y = \dfrac{2}{x}$ and $y = 3 - x$ for $x > 0$.

The shaded area is enclosed between the two graphs and their points of intersection A and B, as shown.



(i) Find the coordinates of the points A and B. **2**

(ii) The shaded area is rotated about the y-axis. Find the exact volume of the solid formed. **3**

End of Examination

Student Number

Mathematics Extension 1 Trial

Section I Answer Sheet

Completely fill the response oval representing the most correct answer.

1.	A ⬭	B ⬭	C ⬭	D ⬭
2.	A ⬭	B ⬭	C ⬭	D ⬭
3.	A ⬭	B ⬭	C ⬭	D ⬭
4.	A ⬭	B ⬭	C ⬭	D ⬭
5.	A ⬭	B ⬭	C ⬭	D ⬭
6.	A ⬭	B ⬭	C ⬭	D ⬭
7.	A ⬭	B ⬭	C ⬭	D ⬭
8.	A ⬭	B ⬭	C ⬭	D ⬭
9.	A ⬭	B ⬭	C ⬭	D ⬭
10.	A ⬭	B ⬭	C ⬭	D ⬭

2020 12MAX TRIAL - HHHS - SOLUTIONS

1. C. 7 letters 3S, 2C

$$\frac{7!}{3!2!}$$

2. D

$$y = \frac{c}{x/c} = \frac{c^2}{x}$$

3. C

if $\begin{bmatrix} a \\ b \end{bmatrix}$ is perpendicular,

then $-2a + 3b = 0$

$$\frac{a}{b} = \frac{3}{2}$$

$\begin{bmatrix} -6 \\ -4 \end{bmatrix}$ satisfies this

4. C

$\sin^2 t = (y-1)^2$, $\cos^2 t = x$.

$\therefore x^2 + (y-1)^2 = 1$

5. B

$\alpha + \beta = \frac{5}{3}$ $\alpha\beta = -2$.

$$\alpha^2 + \beta^2 = (\alpha+\beta)^2 - 2\alpha\beta$$

6. A.

$$= \frac{25}{9} + 4$$

$$= \frac{61}{9}$$

7. B

$y' = y(x-2)$ $x = 2 \Rightarrow y' = 2$

8. D

$$\vec{AB} = (3,7) - (-1,4)$$
$$= \begin{bmatrix} 4 \\ 3 \end{bmatrix}$$
$$|\vec{AB}| = \sqrt{4^2+3^2}$$
$$= 5$$

9. A.

10. C

D _ _ _ $3 \times 2!$

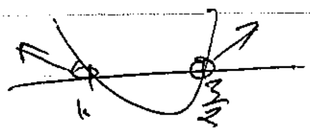
_ D _ _ $2 \times 2!$

_ _ D A $2!$

12

11 a

$$\frac{x}{x-1} \leq 3$$
$$x(x-1) \leq 3(x-1)^2$$
$$3(x-1)^2 - x(x-1) \geq 0$$
$$(x-1)\{3x-3-x\} \geq 0$$
$$(x-1)(2x-3) \geq 0$$



$$\therefore x < 1,\ x \geq \frac{3}{2}$$

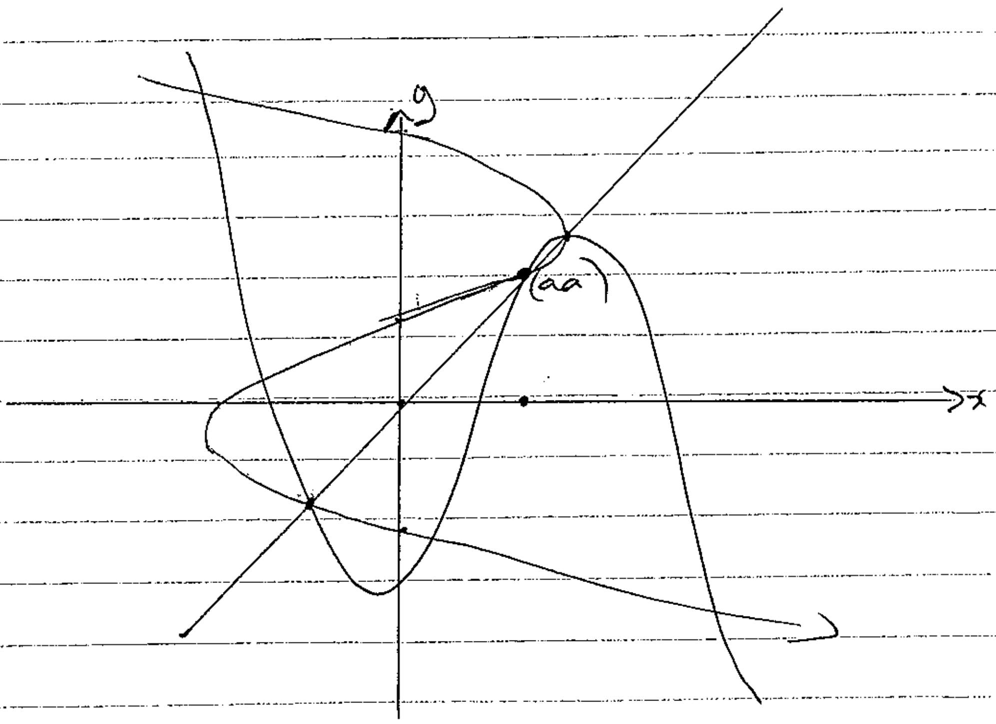
b $|2x+3| \leq 7$

$-7 \leq 2x+3 \leq 7$

$-10 \leq 2x \leq 4$

$-5 \leq x \leq 2$

c.)



d) i) $x^3+3Ax^2-4A=0$, $\alpha, \alpha+\beta, \beta$

$\alpha+\beta+(\alpha+\beta) = \frac{-3A}{1}$

$\therefore 2(\alpha+\beta) = -3A$

$\alpha+\beta = -\frac{3}{2}A$

ii) $\alpha\beta+\alpha(\alpha+\beta)+\beta(\alpha+\beta) = \frac{0}{1}$

$\alpha\beta+(\alpha+\beta)^2 = 0$

$\alpha\beta = -(\alpha+\beta)^2$

$= -\frac{9}{4}A^2$

iii) $\alpha\beta(\alpha+\beta) = -4A$

$-\frac{9}{4}A^2 \times \left(-\frac{3}{2}A\right) = -4A$

$-\frac{27}{8}A^3 = -4A$

$-27A^2 = -36, \quad A>0$

$A^2 = \frac{36}{27} \qquad \frac{32}{27}$

$\therefore A = \frac{6}{3\sqrt{3}} \qquad \frac{4\sqrt{2}}{3\sqrt{3}}$

$= \frac{2}{\sqrt{3}}$

e) i) $\int \frac{-1}{\sqrt{\frac{1}{4}-x^2}}\,dx = \cos^{-1}2x + c$

ii) $\int \sin^2 2x\,dx = \frac{1}{2}\int(1-\cos 4x)\,dx \qquad \cos 2x = 1-2\sin^2 x$

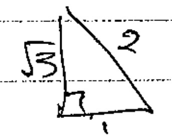
$= \frac{1}{2}\left(x - \frac{\sin 4x}{4}\right) + c$

Q12.

a) $I = \int_{\frac{\pi}{6}}^{\frac{\pi}{3}} \frac{3\cos x}{1+\sin x}\,dx$

let $u = \sin x \quad du = \cos x\,dx.$

at $x = \frac{\pi}{3},\ u = \frac{\sqrt{3}}{2}$



$x = \frac{\pi}{6},\ u = \frac{1}{2}$

$I = 3\int_{\frac{1}{2}}^{\frac{\sqrt{3}}{2}} \frac{du}{1+u}$

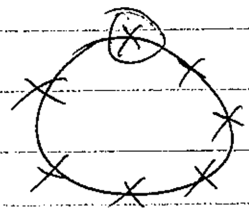
$= 3\Big[\ln|1+u|\Big]_{\frac{1}{2}}^{\frac{\sqrt{3}}{2}}$

$= 3\left(\ln\left(1+\frac{\sqrt{3}}{2}\right) - \ln\left(1+\frac{1}{2}\right)\right)$

$= 3\ln\left(\frac{2+\sqrt{3}}{3}\right)$

$= 0.6550368$

b) i)

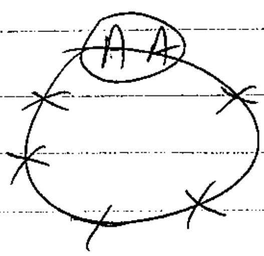


7 peoples at table.

fix one $\Rightarrow$ arrangements $= 6!$

$= 720$

ii)

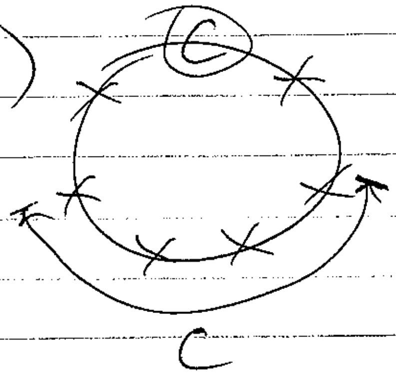


6 groups $\Rightarrow 5!$

2 adults $\Rightarrow 2!$

arrangements $= 5!.2!$

$= 240$

iii) 

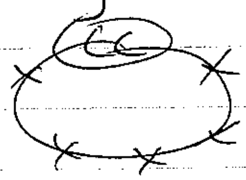
fix one child.

other child sits in one of four spots

remaining $\Rightarrow 5!$

$\therefore$ arrangements $= 5! \times 4$
$= 480$

alternatively



arrangements $= 720 - 5!.2!$
$= 480$

c) i) $\cos x - 2\sin x = 1, \ -\pi \le x \le \pi$

$\cos x = \frac{1-t^2}{1+t^2}, \ \sin x = \frac{2t}{1+t^2}$

$\therefore \frac{1-t^2}{1+t^2} - 2 \times \frac{2t}{1+t^2} = 1$

$\cancel{1} - t^2 - 4t = \cancel{1} + t^2$

$\therefore 2t^2 + 4t = 0$

$t^2 + 2t = 0$

ii) $t(t+2) = 0$

$\therefore t = 0, -2$

$\tan\frac{x}{2} = 0, -2$

$\therefore \frac{x}{2} = \tan^{-1}0, \tan^{-1}(-2)$

$$x = 0,\ 2\tan^{-1}(-2)$$
$$= 0,\ -2.214297$$

d) i) $\frac{dV}{dt} = -k\sqrt{V}$

$$\int \frac{dV}{V^{\frac{1}{2}}} = \int -k\,dt$$

$$2V^{\frac{1}{2}} = -kt + c$$

$$V = \frac{1}{4}(c-kt)^2$$

ii) $t=0,\ V=30,\ \frac{dV}{dt} = 1.8$

$$30 = \frac{1}{4}(c - k(0))^2$$
$$c = 2\sqrt{30}$$

$$-1.8 = -k\sqrt{30}$$
$$k = \frac{9}{5\sqrt{30}}$$

$$\therefore\ V = \frac{1}{4}\left(2\sqrt{30} - \frac{9}{5\sqrt{30}}t\right)^2$$

for $V=0$ $\quad 2\sqrt{30} - \frac{9}{5\sqrt{30}}t = 0$

$$t = 2\sqrt{30} \times \frac{5\sqrt{30}}{9}$$
$$= 33\tfrac{1}{3}\text{min}$$

Q13.

a) $\frac{dS}{dt} = 0.8\,m^2/s$, $S = 4\pi r^2$, find $\frac{dr}{dt}$

$$\frac{dS}{dr} = 8\pi r.$$

$$\frac{dr}{dt} = \frac{dr}{dS} \cdot \frac{dS}{dt}$$
$$= \frac{1}{8\pi r} \cdot 0.8$$

at $r = 2.3$ $\frac{dr}{dt} = \frac{0.8}{8\pi(2.3)}$
$$= 0.0138396 \ ms^{-1}$$

b) $a + ar + ar^2 + \ldots + ar^{n-1} = \frac{a(r^n - 1)}{r-1}$, $n \geq 1$

Prove true for $n=1$

LHS $= a$ RHS $= \frac{a\cancel{(r-1)}}{\cancel{r-1}}$
$= a =$ LHS

$\therefore$ true for $n=1$

Assume true for $n=k$

$\therefore a + ar + ar^2 + \ldots + ar^{k-1} = \frac{a(r^k - 1)}{r-1}$ induction hypothesis

Prove true for $n=k+1$

i.e. $a + ar + ar^2 + \ldots + ar^{k-1} + ar^k = \frac{a(r^{k+1} - 1)}{r-1}$

$$LHS = a + ar + ar^2 + \dots + ar^{k-1} + ar^k$$

$$= \frac{a(r^k - 1)}{r-1} + ar^k \quad \text{using hypothesis}$$

$$= a\left[\frac{r^k - 1}{r-1} + r^k\right]$$

$$= a\left\{\frac{r^k - 1 + r^k(r-1)}{r-1}\right\}$$

$$= a\left\{\frac{\cancel{r^k} - 1 + r^{k+1} - \cancel{r^k}}{r-1}\right\}$$

$$= \frac{a(r^{k+1} - 1)}{r-1}$$

$$= RHS.$$

$\therefore$ by principle of mathematical induction statement is true for all $n \geq 1$.

c) i)

$$\cos 3\theta = \cos(2\theta + \theta)$$
$$= \cos 2\theta \cos\theta - \sin 2\theta \sin\theta$$
$$= (2\cos^2\theta - 1)\cos\theta - 2\sin\theta\cos\theta\sin\theta$$
$$= 2\cos^3\theta - \cos\theta - 2\sin^2\theta\cos\theta$$
$$= 2\cos^3\theta - \cos\theta - 2(1 - \cos^2\theta)\cos\theta.$$
$$= 2\cos^3\theta - \cos\theta - 2\cos\theta + 2\cos^3\theta$$
$$= 4\cos^3\theta - 3\cos\theta$$

ii)

$$8x^3 - 6x = 1$$

let $x = \cos\theta$

$\therefore\ 8\cos^3\theta - 6\cos\theta = 1$

$2(4\cos^3\theta - 3\cos\theta) = 1$

$\therefore 2\cos 3\theta = 1$

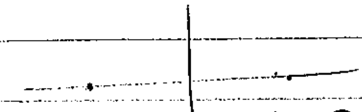
$\cos 3\theta = \frac{1}{2}$

$-\pi \leq \theta \leq \pi$

$-3\pi \leq 3\theta \leq 3\pi$

$\cos 3\theta > 0$ 3θ in 1^{st} + 4^{th} quadrants

as $\cos\frac{\pi}{3} = \frac{1}{2}$



then $3\theta = -2\pi - \frac{\pi}{3}, -2\pi + \frac{\pi}{3}, -\frac{\pi}{3}, \frac{\pi}{3}, 2\pi - \frac{\pi}{3}, 2\pi + \frac{\pi}{3}$

$\theta = -\frac{7\pi}{9}, -\frac{5\pi}{9}, -\frac{\pi}{9}, \frac{\pi}{9}, \frac{5\pi}{9}, \frac{7\pi}{9}$

$x = \cos\left(\frac{\pi}{9}\right), \cos\left(\frac{5\pi}{9}\right), \cos\left(\frac{7\pi}{9}\right)$

$\cos x$ is an even function

d) i) $\overrightarrow{OX} = \frac{1}{2}\underset{\sim}{c} + \frac{1}{2}\underset{\sim}{a}$

$= \frac{1}{2}(\underset{\sim}{a} + \underset{\sim}{c})$

ii) $\overrightarrow{OD} = \frac{k+1}{k}\overrightarrow{OC} = \frac{k+1}{k}\underset{\sim}{c}$

iii) $\overrightarrow{XD} = \overrightarrow{OD} - \overrightarrow{OX}$

$= \frac{k+1}{k}\underset{\sim}{c} - \frac{1}{2}(\underset{\sim}{a} + \underset{\sim}{c})$

$= \left(\frac{k+1}{k} - \frac{1}{2}\right)\underset{\sim}{c} - \frac{1}{2}\underset{\sim}{a}$

$= \frac{2k+2-k}{2k}\underset{\sim}{c} - \frac{1}{2}\underset{\sim}{a}$

$$= \frac{k+2}{2k}\underset{\sim}{c} - \frac{1}{2}\underset{\sim}{a}$$

as $\frac{k+2}{2k}\underset{\sim}{c} - \frac{1}{2}\underset{\sim}{a} = 3\underset{\sim}{c} - \frac{1}{2}\underset{\sim}{a}$

$$\frac{k+2}{2k} = 3 \quad \text{(equating } \underset{\sim}{c} \text{ component)}$$

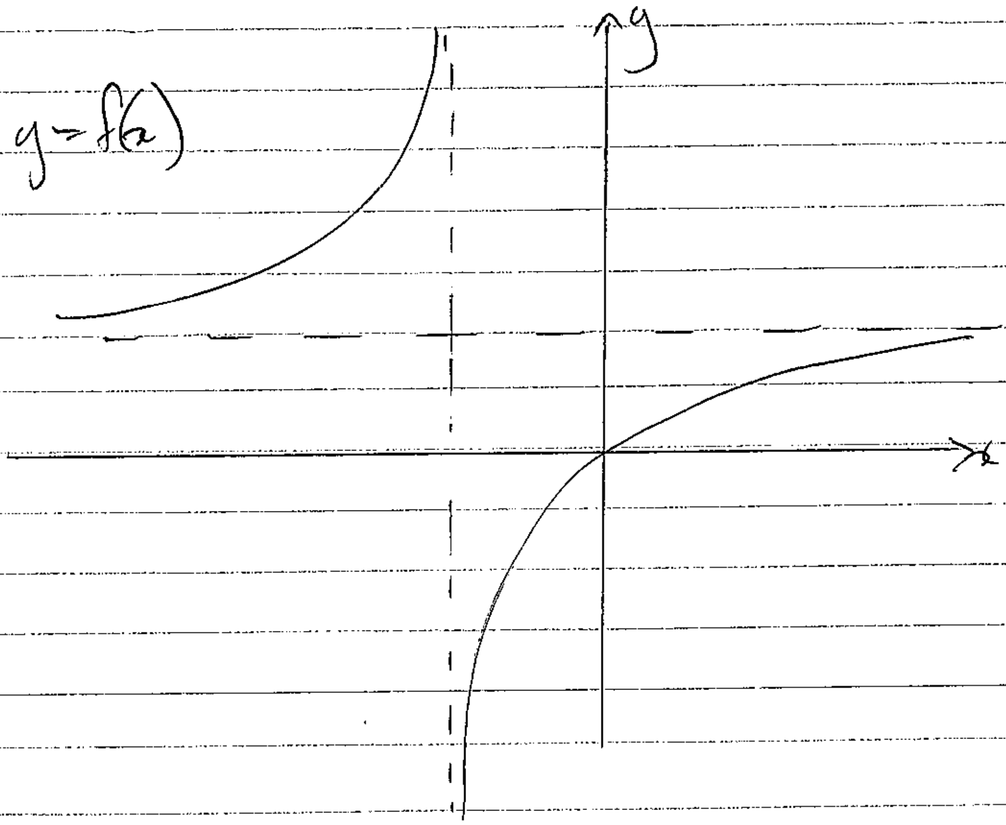
$$k+2 = 6k$$

$$5k = 2$$

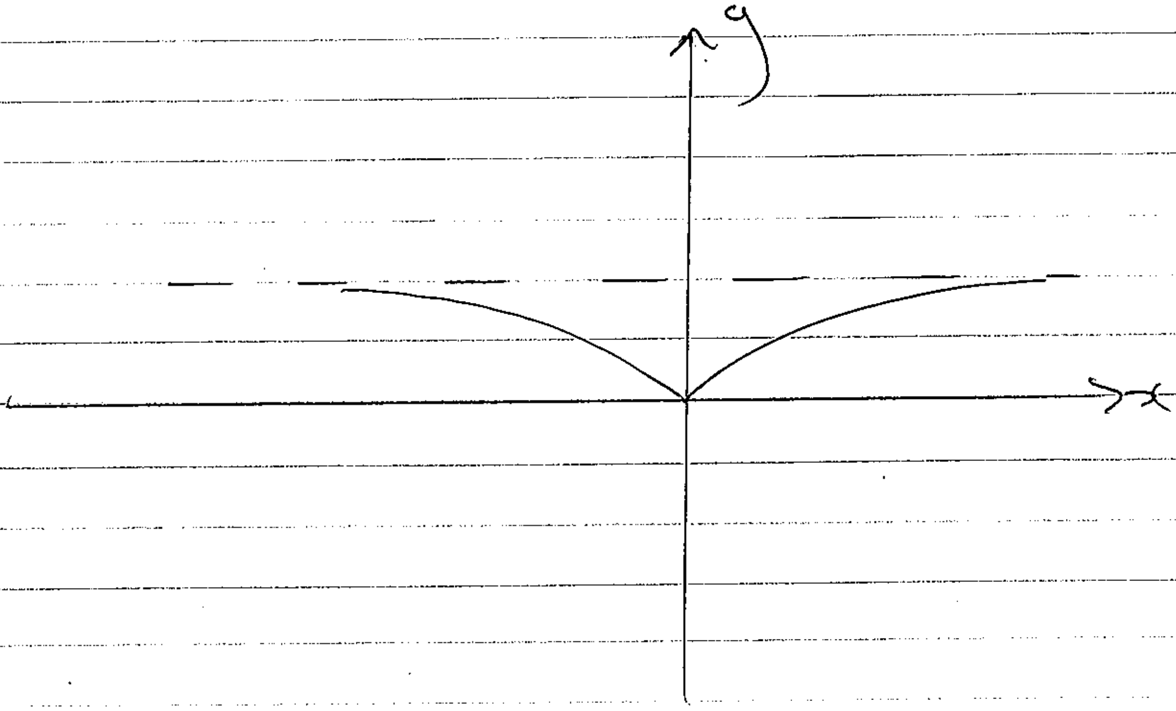
$$k = \frac{2}{5}$$

Q14

a) i) $y = f(x)$



ii)



b) i) number of arrangements $= 4^4$
$= 256$

ii) ways that 4 meals can be had

$\underline{3}\ \underline{1}\ \underline{\ }\ \underline{\ }$

$\underline{2}\ \underline{2}\ \underline{\ }\ \underline{\ }$

for 3/1 $\quad {}^4C_3 \times 2!$

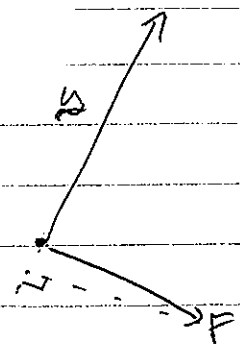
for 2/2 $\quad {}^4C_2 \times 2!$ $\qquad$ 14641

total for these two meals $= {}^4C_3 \times 2! + {}^4C_2 \times 2!$
$= 20.$

combinations of two meats $= {}^4C_2$

$$\text{Probability} = \frac{20 \times {}^4C_2}{256}$$
$$= \frac{5}{16}$$

c) i) $F = \begin{bmatrix} 3 \\ -2 \end{bmatrix}$

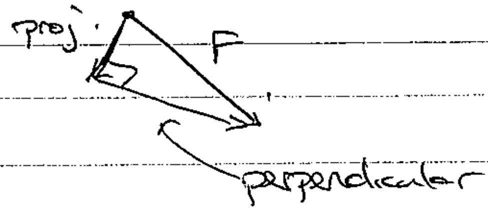


unit vector in direction of l.

$$\hat{u} = \frac{1}{\sqrt{29}} \begin{bmatrix} 2 \\ 5 \end{bmatrix}$$

$$\text{proj}_u F = (F \cdot \hat{u})\,\hat{u}$$
$$= \left(3 \times \frac{2}{\sqrt{29}} + -2 \times \frac{5}{\sqrt{29}}\right) \cdot \frac{1}{\sqrt{29}} \begin{bmatrix} 2 \\ 5 \end{bmatrix}$$
$$= \frac{-4}{29} \begin{bmatrix} 2 \\ 5 \end{bmatrix}$$

ii)



perpendicular component

$$= F - \text{proj}_u F$$

$$= \begin{bmatrix} 3 \\ -2 \end{bmatrix} - \frac{4}{29} \begin{bmatrix} 2 \\ 5 \end{bmatrix} = -\frac{1}{29} \begin{bmatrix} 5 \\ 22 \end{bmatrix}$$

iii) opposite of parallel component force applied to object. is

$$\frac{4}{29}\begin{bmatrix}2\\5\end{bmatrix}$$

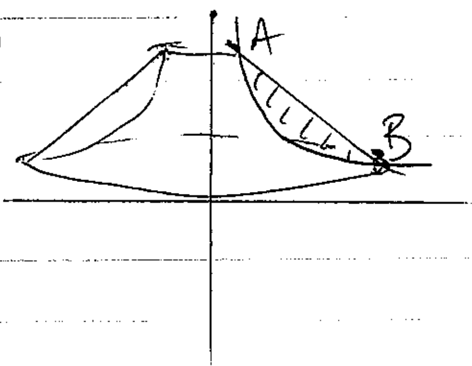
d) i) $y=\frac{2}{x}$, $y=3-x$.

equating $\frac{2}{x}=3-x$

$$x^2-3x+2=0$$
$$(x-2)(x-1)=0$$
$$\therefore x=1,2$$

$\therefore$ A is $(1,2)$ and B is $(2,1)$

ii)



$$V=\pi\int_1^2\left((3-y)^2-\left(\frac{2}{y}\right)^2\right)dy$$
$$=\pi\int_1^2\left((3-y)^2-\frac{4}{y^2}\right)dy$$
$$=\pi\left[\frac{(3-y)^3}{-3}+\frac{4}{y}\right]_1^2$$
$$=\pi\left(\frac{1^3}{-3}+\frac{4}{2}-\left(\frac{2^3}{-3}+\frac{4}{1}\right)\right)$$
$$=\pi\left(2-\frac{1}{3}-4+\frac{8}{3}\right)$$
$$=\frac{\pi}{3}\text{ units}^3$$